Exam Elementary Particles
June 19, 2019
Start: 14:00h End: 17:00h

Each sheet ***with your name and student ID***

INSTRUCTIONS: This is a closed-book and closed-notes exam. You are allowed to bring one A4 page written by you on both sides, with useful formulas. The exam duration is 3 hours. There is a total of 9 points that you can collect. Use the official exam paper for *all* your work and ask for more if you need.

1. Short Essay/Questionnaire (4 points) Consider the Lagrangian density of a QCD-like massless theory, i.e., the $SU(N)$ Yang-Mills theory coupled to N_f flavors of massless Dirac fermions in the fundamental representation of $SU(N)$:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{f=1}^{N_f} \bar{\psi}^f_i \left(i\partial\!\!\!/ \delta_{ij} + g A\!\!\!/^a T^a_{ij} \right) \psi^f_j \tag{1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{2}$$

is the gauge field strength tensor, f^{abc} are the structure constants of $SU(N)$, T^a, $a = 1, \dots, N^2 - 1$ are the generators of $SU(N)$ in the fundamental representation of $SU(N)$, the indices i, j are color indices, $i, j = 1, \dots, N$, and f is a flavor index, $f = 1, \dots, N_f$.

The beta function to 2 loops in perturbation theory reads:

$$\beta(g) = -\frac{dg}{d \log z} = -\beta_0 g^3 - \beta_1 g^5 + \dots$$

where z is a length scale and the coefficients $\beta_{0,1}$:

$$\begin{aligned} \beta_0 &= \frac{1}{(4\pi)^2} \left(\frac{11}{3} N - \frac{2}{3} N_f \right) \\ \beta_1 &= \frac{1}{(4\pi)^4} \left(\frac{34}{3} N^2 - \frac{13}{3} N N_f + \frac{N_f}{N} \right) \end{aligned} \tag{3}$$

are universal, i.e., renormalization-scheme independent.

Discuss asymptotic freedom by answering briefly (a few sentences per question, formulas and graphs as needed) the following questions:

1. What is asymptotic freedom, and how does it manifest itself in the beta function, $\beta(g)$, of the theory?
2. For which value of N_f is asymptotic freedom lost, and why?
3. Based on the 2-loop beta function, can the theory also develop a nontrivial, i.e., interacting infrared (IR) fixed point? Show your argument.
4. Find the leading order ultraviolet (UV) asymptotic behavior of the running coupling, $g(x)$, by integrating the 1-loop beta function between a length scale μ^{-1} and a length scale x.

2. (2 points) Use dimensional analysis and considerations based on the superficial degree of divergence of 1PI and primitively divergent Feynman diagrams to conclude that the Higgs sector of the Standard Model in the unbroken phase, with Lagrangian density:

$$\mathcal{L}_H = D_\mu H^\dagger D^\mu H - \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 \tag{4}$$

with $\mu^2 > 0$ and $\lambda > 0$ and covariant derivative:

$$D_\mu H = (\partial_\mu - i\frac{g}{2}W_\mu^a \tau^a - i\frac{g'}{2}B_\mu)H \tag{5}$$

with $H(x)$ the $SU(2)_L$ doublet

$$H(x) = \begin{pmatrix} h^+(x) \\ h^0(x) \end{pmatrix} \tag{6}$$

is renormalizable in $d = 4$ spacetime dimensions.

Hints:

- Eq. (4) is the Lagrangian of a ϕ^4 theory with a gauge symmetry, thus coupled to spin-1 gauge bosons via the covariant derivative, analogously to scalar-QED.
- The superficial degree of divergence is:

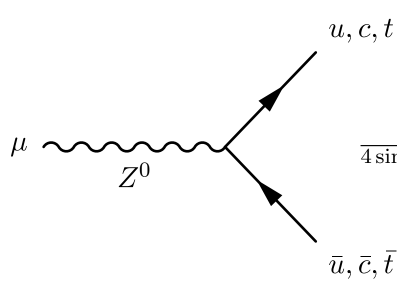
$$D = 4 - E_H - E_G - [g]V_3 - [g^2]V_4 - [\lambda]V_\lambda \tag{7}$$

  where [.] indicates the (energy) dimension of its argument and $[g] = [g']$, E_H is the number of external Higgs lines, E_G the number of external gauge-boson lines, V_3 the number of vertices of type gauge-Higgs-Higgs, V_4 the number of vertices of type gauge-gauge-Higgs-Higgs and V_λ the number of Higgs self-interaction vertices.

3. (3 points) In the Standard Model the neutral weak gauge boson Z^0 can decay into a pair of a quark (q) and an antiquark ($\bar{q}$), where q can be any of the up or down quarks, $q = u, d, c, s, t, b$.

The Feynman rules for the vertices that mediate the decay at tree level differ for the up quarks (u, c, t) and the down quarks (d, s, b).

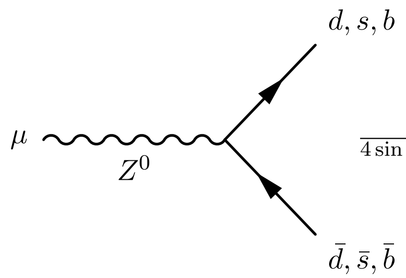
For the up quarks one has:



$$\frac{ie}{4\sin\theta_W \cos\theta_W}\gamma^\mu(a_u - \gamma_5)$$

with $a_u = 1 - \frac{8}{3}\sin^2\theta_W$.

For the down quarks one has:



$$\frac{-ie}{4\sin\theta_W\cos\theta_W}\gamma^\mu(a_d - \gamma_5)$$

with $a_d = 1 - \frac{4}{3}\sin^2\theta_W$.

Note that these interactions do not mix flavors nor do they mix up with down quarks.

Neglecting all quark masses, show that the unpolarized squared amplitude that enters the total decay rate $\Gamma_q = \sum_q \Gamma(Z^0 \to \bar{q}q)$ at tree level is given by:

$$X = \frac{e^2 M_Z^2}{4\sin^2\theta_W\cos^2\theta_W}(2 + a_u^2 + a_d^2) \tag{8}$$

Useful formulas:

- $(\bar{u}\Gamma v)^\dagger = \bar{v}\gamma^0\Gamma^\dagger\gamma^0 u$, with Γ any product of γ matrices.
- $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\epsilon^{\mu\nu\rho\sigma}$ antisymmetric!
- $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$
- The energy projectors of a spin-1/2 massless Dirac spinor are given by:

$$\sum_{r=1,2} u_r(\vec{p})\bar{u}_r(\vec{p}) = \not{p} \tag{9}$$

- The sum over the polarizations of the massive vector particle Z^0 is given by:

$$\sum_{a=1}^{3}\epsilon_a^\mu(k)\epsilon_a^\nu(k) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M_Z^2}$$

with ϵ_μ real. This is not QED, $k^\mu k^\nu$ terms matter!